Monetary Equilibrium

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Abstract

One implication of the concept of monetary equilibrium is that the money supply should vary with money demand. In a recent paper, Bagus and Howden (2011) argue that this conclusion is predicated on the assumption of price stickiness. The purpose of this paper is to suggest that the foundation of monetary equilibrium is the role of money as a medium of exchange. As such, changes in the demand for money result in changes in both nominal and real spending that are welfare-reducing. This proposition is then used to examine whether a monetary policy in which the central bank varies the money supply in response to money demand can be considered optimal. In addition, the paper considers how a free banking system with competitive note issuance would vary the money supply in response to changes in money demand. In both cases, the results are consistent with the concept of monetary equilibrium. In addition, these results can be obtained even when prices are perfectly flexible if trade is decentralized (i.e. not conducted in Walrasian markets). Price stickiness is therefore not a necessary condition to suggest that the money supply should vary with money demand.

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1 Introduction

The concept of monetary equilibrium is one that permeates a broad spectrum of monetary theory.[1] Put succinctly, monetary equilibrium describes the condition under which actual money balances are equal to desired money balances. According to the theory of monetary equilibrium, deviations between actual and desired money balances have adverse consequences on market welfare. For Monetarists, the deviations result in a decline in nominal spending and real output. For Austrians and coordination Keynesians, such deviations result in a divergence between the market and natural rate of interest that, in the Austrian case, leads to a misallocation of capital. A common conclusion that follows from this concept is that the money supply should vary with money demand so as to maintain the stability of nominal income and the equilibrium interest rate.

In a recent paper, Bagus and Howden (2011) challenge the idea consistent with monetary equilibrium theory that the money supply should vary with money demand. The authors argue that such a conclusion is based on what they perceive as the fundamental foundation of monetary equilibrium theory, namely price stickiness.[2] Subsequently, they argue that price stickiness is a weak reed upon which to rest this normative conclusion. Rather than address the issue of price stickiness directly, the purpose of this paper is to provide an explicit account of the concept of monetary equilibrium and to consider how a central bank or a free banking system should operate within that framework.

The central foundation of monetary equilibrium theory is the role of money as medium of exchange, not price stickiness. Money's unique role as medium of exchange means that changes in the demand for money will reverberate throughout all markets. As a result, increases in the demand for money will cause reductions in spending across markets. Further, it is the position of monetary equilibrium theory that these reductions in spending are welfare-reducing and that corresponding changes in the money supply, whether by a central bank or through competitive note issuance, can improve welfare. While it is true that a number of previous proponents of monetary equilibrium

[1] This is true, for example, of American Monetarists (Warburton, 1950; Friedman 1974; Friedman and Schwartz, 1982; Yeager, 1986), coordination Keynesians (Clower, 1965, 1969; Leijonhufvud, 1968), and Austrians (Horwitz, 2000; White, 1999). For a historical overview of monetary equilibrium, see also Selgin (1988, Ch. 4).

[2] This is only one argument put forth by Bagus and Howden. The second argument is that the policy implied by monetary equilibrium theory might lead to intertemporal discoordination consistent with Austrian business cycle theory. On this point, see Bagus and Howden (2012). Capital is absent from the present paper and therefore the present analysis cannot assess this claim. However, this is a fruitful area for future research.

have relied on the assumption of sticky prices to generate this conclusion, it will be shown below that sticky prices are not a necessary condition for the conclusion. The assumption of sticky prices is necessary only to the extent that one relies on the assumption of Walrasian markets.

In what follows, a monetary search model is employed to examine the role of monetary equilibrium. The use of this framework is important because it provides an essential role for money in the exchange process, a definition for the determination of observed economic aggregates such as velocity and nominal income, and an explicit measure of market welfare. The model is therefore capable of considering the effects of aggregate money demand shocks on nominal spending and market welfare. Given the basic structure and implications of model, the role of central banks and competitive note issuance is considered. In the case of the central bank, a nominal income target in which the money supply is adjusted based on changes in money demand is considered to be an optimal policy. In addition, it is shown that under free banking with competitive note issuance, banks will vary the money supply with the demand for money. Throughout the model, prices are flexible. Thus, price stickiness is not a necessary condition for advocating a free banking system in which the notes in circulation vary with the demand for money nor a monetary policy that varies the money supply in response to fluctuations in money demand. Rather, it is the existence of decentralized trade, or non-Walrasian markets, that generates these results.

2 A Lightspeed Tour of Monetary Equilibrium

As noted above, the concept of monetary equilibrium is one that permeates through seemingly divergent schools of monetary economic thought. Central to this concept is the role of money as medium of exchange. In particular, this role implies that "goods buy money, and money buys goods – but goods do not buy goods in any organized market" (Clower, 1969). As a result, a ceteris paribus change in the supply or demand for money has implications for the markets for *all* goods and services.

The concept of monetary equilibrium is key to the analysis of Friedman (1974) and Friedman and Schwartz (1963a, 1963b, 1982) and is perhaps where the concept attains its most familiar exposition. According to the quantity theory of nominal income, as developed in Friedman (1974) as the theoretical underpinning of his empirical work with Anna Schwartz, the source of fluctuations in nominal income are deviations of real money balances and nominal money balances, where the former represents an explicit account of money demand and the latter the variation in the supply of money. In other words, it is the deviation between actual and desired money balances that cause fluctuations in spending and ultimately nominal income. Friedman is known frequently to address this concept in the context of the equation of exchange, $MV = PY$. The use of the equation of exchange is often criticized as tautological. Indeed, Friedman (1974) argued that "the tautology embodied in the quantity equation is a useful device for clarifying the variables stressed in the quantity theory." Friedman notes that, "the quantity theory is not, however, this tautology." The quantity theory instead represents an analysis of the interaction between the supply of and demand for money based on an explicit account of the latter. The equation of exchange is a "useful tautology" because it communicates the idea that deviations of actual money balances (embodied in M) and desired money balances (embodied in V) are the primary cause of fluctuations in nominal income.[3]

This insight also forms the basis for the dual-decision hypothesis of Clower (1963). According to this view, planned purchases are conditional on planned sales; this is the idea of notional demand. If the demand for nominal money balances at a given price level exceeds the nominal supply of money, agents will try to accumulate money balances and do so through a reduction in spending. The reduction in spending means that sellers will now have a harder time finding buyers and there is a corresponding reduction in sales. Since planned purchases are conditional on planned sales, it follows that a reduction in current sales will constrain current purchases; effective demand is lower than notional demand.

The characteristic described by Clower is unique to a monetary economy as the conditions that give rise to money similarly give rise to the potential for a coordination failure. For example, consider a case of multilateral barter in which there are three agents, three goods, and an absence of a double coincidence of wants between each pairwise combination of agents. If each agent demands

[3]Yeager (1994: 158) discusses the role of useful tautologies in economics and notes notes that the equation of exchange is an example of a tautology able to "illustrate certain relations between definitional truths and empirical reality." This is clearly consistent with Friedman's description of the equation of exchange as a "useful tautology." The fact that the equation of exchange is a tautology is of little consequence in the context in which Friedman, and others, use it. For example, as Yeager notes, one could write down a "chairs" version of the equation of exchange. The equation of exchange will be valid in this form just as in the money version. However, money and chairs have different functions and therefore "the money version of the equation of exchange has a usefulness that the chairs version lacks" Yeager (1994: 160).

something produced by another agent, it is possible under these circumstances for a multilateral agreement to exist in which agent 1 promises to give good 1 to agent 2 on the promise that agent 2 will provide good 2 to agent 3 and agent 3 will provide good 3 to agent 1.[4] Nonetheless, as first noted by Ostroy (1973), each agent has the incentive to produce less than promised and inform the buyer (erroneously) that they had received less than promised thereby generating a larger gain from trade. If other agents know this is the case, and cannot access trading history of other agents, trade will likely not take place. Money therefore serves an essential role as medium of exchange because "money is memory" (Kocherlakota, 1998). If each agent presents money to the other in exchange for goods this replaces the need (and the cost) of maintaining records of trading history.

Deviations of the supply and demand for money balances create a coordination problem precisely because money is a substitute for information and because sales can be held in the form of money. The reduction in trade as a result of the deviation between actual and desired money balances could be eliminated if the supply of money was greater. However, economic agents lack the ability to signal to others that this is the case and, in the absence of private note issuance, a mechanism by which to facilitate such trade even if such a signal did exist.[5]

Broadly, the implications of monetary equilibrium are as follows. A reduction in trade as a result of an excess demand for money is welfare-reducing in that there is a Pareto-improving allocation. As a result, it is important to consider monetary arrangements that would potentially be Pareto-improving. Typically, in terms of monetary policy, advocates of the monetary equilibrium framework have advocated the stabilization of nominal income.[6] An alternative approach, advocated by Selgin (1988) and others, is that a free banking system in which banks are free from regulation and are able to issue their own notes would vary note issuance with the demand for money as a result of profit motives thereby promoting monetary equilibrium.

[4]This is the basic Wicksellian triangle (Wicksell, 1934).

[5]Note that in a barter economy trades are based on an exchange of goods for goods and therefore a reduction in sales is the result of changes in real factors (e.g. lower productivity, a bad harvest, changes in preferences toward or away from more autarkic production), which are immediately communicated via trade with corresponding relative price adjustments. Money substitutes for record-keeping, but in doing so enables individuals to hold the proceeds from trade in the form of money.

[6]The stabilization of nominal income is equivalent in the language of the equation of exchange to the stabilization of MV. Selgin (1988) contains references to such advocacy on the part of early monetary equilibrium theorists.

3 A Model of Monetary Equilibrium

Any meaningful analysis of monetary equilibrium theory must begin with the premise that money is important as a medium of exchange. In other words, money is essential for trade. As a result, the formal model represented in this paper is based on that in Lagos and Wright (2005). The structure of the model is as follows. Time, $t = 1, 2, \ldots, \infty$, is discrete. Each economic agent is infinitely-lived. Each period is divided into two sub-periods. What distinguishes each sub-period is the nature of trade. In the first sub-period, trade is decentralized, economic agents meet pairwise and they negotiate terms of trade. In the second sub-period, trade is centralized in that individuals meet in an organized market and are able to trade and re-allocate their portfolio. Economic agents are heterogeneous. Specifically, it is assumed that there are two types of agents, designated by their role in the first sub-period. The first type of agent, a buyer, wants to consume in the decentralized market (DM), but does not produce anything to sell in that market. Buyers produce to sell in the centralized market (CM). The second type of agent, sellers, produce to sell in the DM, but do not produce in the CM. Sellers use the proceeds from their sales in the DM to consume in the CM. Thus, given the heterogeneity of buyers and sellers, there is a basic absence-of-double-coincidence-of-wants problem and therefore there is a role for money. Hereafter, the good produced by the seller is referred to as the search good and the good produced by the buyer is general good. Both goods are non-storable. Fiat money exists, is storable, and is intrinsically worthless. There is no public record-keeping of trading histories and therefore money is essential.[7] Prices are perfectly flexible. There is a central bank that controls the money supply through lump-sum transfers, $T_t = M_t - M_{t-1}$, to buyers in the DM.

There are a continuum of buyers and sellers each with unit mass. Buyers have preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(q_t) + x_t]$$

where β is the discount factor, $u(q)$ is the utility generated by consuming the search good, q, satisfying $u' > 0$, $u'' < 0$, $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and x_t is the quantity consumed of the general good.

[7] Money is essential in the sense that it allows agents to engage in trade that would not be possible in its absence. In other words, money expands the set of feasible trades. See Kocherlakota (1998).

Sellers have preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [-c(q_t) + x_t]$$

where $c(q)$ is the cost, measured in terms of disutility, of producing the search good and satisfies $c' > 0$, $c'' \geq 0$, $c(0) = 0$.

Finally, define ϕ as the goods price of money and assume that $\phi_t \geq \beta\phi_{t+1}$.[8] This assumption assures that a monetary equilibrium exists because money is costly to hold. If this condition did not hold, individuals would seek to accumulate infinite money balances.

The analysis proceeds by starting in the CM and working backward through the DM. Equilibrium is summarized by a two-equation system that solves for the equilibrium quantity of the search good and nominal income, taking the money supply as exogenously determined. The subsequent analysis then examines the behavior of the money supply in the context of free banking and in the context of central banking.

3.1 The Centralized Market

Buyers bring a quantity of real money balances into the CM, ϕm, where ϕ is defined as the goods price of money. In the CM, buyers produce the general good, one-for-one using their labor, h.[9] Buyers consumer some of the general good, x, and have real money balances at the end of trading in the centralized market, $\phi m'$. At time t, a buyer's budget constraint is given by:

$$\phi_t m' + x_t = h_t + \phi_t m \tag{1}$$

The value function for a buyer holding m units of money at the beginning of the CM is:

$$W_t^b(m) = \max_{m', x_t, h_t} \left(x_t - h_t + \beta V_{t+1}^b(m') \right) \tag{2}$$

where V^b is the value function for the buyer holding money balances, m', entering the DM in the

[8] Note that this is the goods price of money. This is not the money price of goods. In other words, $\phi = 1/P$ where P is the price level. This notation is used because it is possible for an equilibrium to exist in which money has no value.

[9] This assumption simplifies the model because it implies that $w = 1$ where w is the wage.

next period. Assuming an interior solution, one can solve equation (1) for h_t and substitute this into equation (2) such that:

$$W_t^b(m) = \phi_t m + \max_{m'} \Big[-\phi_t m' + \beta V_{t+1}^b(m') \Big] \tag{3}$$

The first-order condition yields

$$\phi_t = \beta V'_{t+1}(m') \tag{4}$$

Note that the sellers value function is the same as the buyers value function. However, since money is costly to hold, sellers will not carry any money balances in the DM. As a result, we can neglect the analysis of the sellers value functions because they will have the same value each period.

3.2 The Decentralized Market

Buyers enter the DM with money balances, m. In the DM, buyers and sellers meet pairwise and exchange goods for money. All sellers are present in the market whereas a fraction of buyers, α_t, enter the market. A buyer can only meet one seller and therefore buyers and sellers are matched with probability α_t. When they meet, a buyer gives a seller a quantity of real money balances, $\phi_t d$, in exchange for a quantity of the search good, q, where $d \leq m$. The terms of trade are determined by bargaining between the buyers and the sellers.

Here, α_t is assumed to be a stochastic preference shock with mean α and finite variance and measures the degree of trading frictions in the DM. In particular, α_t is isomorphic to an aggregate money demand shock.[10] The justification for such an interpretation is as follows. All buyers that enter the market solve the same optimization problem and therefore hold the same level of money balances, m. Given that buyers have unit mass, this formulation implies that the aggregate money stock is divided between those buyers matched with sellers in the DM and those who are not:

$$M = \alpha m + (1 - \alpha) m$$

Intuitively, α_t, can be thought to represent a shock to money demand because the unique role of money as a medium of exchange means that fluctuations in money demand reverberate through all

[10]See the appendix.

markets. As a result, when money demand increases, it is harder for sellers to find buyers; there is a reduction in α. When money demand decreases, it is easier for sellers to find buyers; there is an increase in α. Put differently, a decrease in α implies that less money balances are held by those who trade in the decentralized market. Similar to a traditional money demand shock, this implies that a larger amount of money balances are held as a store of value.

Given the preferences outlined above, the value function for the buyer entering the DM is given as:

$$V_t^b(m) = \alpha_t\big[u[q_t(m)] - \phi_t d\big] + W_t^b(m) \tag{5}$$

where the quantity of the search good consumed is a function of the money balances of the buyer.

Once the buyer and seller meet, they negotiate the terms of trade using Nash bargaining. Assuming that the buyer's bargaining power is θ, the bargaining problem can be written:

$$[u(q_t) - \phi_t(d + T_t)]^\theta[-c(q) + \phi_t(d + T_t)]^{1-\theta}$$

Given that money is costly to hold, the buyer will never carry more money into the DM than is intended for purchase and therefore $d = m$. Taking the first-order condition of the bargaining problem with respect to q_t, one can solve for the quantity of real money balances that buyer will hold:

$$\phi_t m = \frac{\theta u'(q_t)c(q_t) + (1-\theta)u(q_t)c'(q_t)}{\theta u'(q_t) + 1 - \theta} \equiv z(q_t;\theta) \tag{6}$$

Using the fact that $d = m$, one can differentiate equation (5):

$$V_t'(m) = \alpha_t\{[u'(q_t)(\partial q_t/\partial m)] - \phi_t\} + \phi_t \tag{7}$$

From (6), the implicit function theorem implies that $(\partial q_t/\partial m) = \phi_t/z'(q_t;\theta)$. Substituting this into equation (7) yields:

$$V_t'(m) = \phi_t\left\{\alpha_t\left(\frac{u'(q_t)}{z'(q_t;\theta)} - 1\right) + 1\right\}$$

Iterating forward and substituting this expressing into equation (4) yields:

$$\phi_t = \beta\phi_{t+1}\left\{\alpha_{t+1}\left(\frac{u'(q_{t+1})}{z'(q_{t+1};\theta)} - 1\right) + 1\right\} \tag{8}$$

Finally, in equilibrium, the money supply will equal money demanded, such that $\phi_t M_t = z(q_t;\theta)$. As a result, equation (8) can be re-written as:

$$\frac{z(q_t;\theta)}{M_t} = \beta \frac{z(q_{t+1};\theta)}{M_{t+1}} \left\{ \alpha_{t+1} \left(\frac{u'(q_{t+1})}{z'(q_{t+1};\theta)} - 1 \right) + 1 \right\} \tag{9}$$

Where, given $\{M_t, \alpha_t\}$, equation (9) provides a solution for $\{q_t\}$.

3.3 Monetary Equilibrium

It is typical of the analysis of monetary equilibrium theory to represent monetary equilibrium in the context of the equation of exchange. As Friedman (1974) notes, however, the use of the equation of exchange is to simplify the exposition of the quantity theory of nominal income in which deviations of the money supply and money demand cause fluctuations in nominal income. Despite the simple treatment in the form of explanation, such analysis finds its basis in an explicit treatment of money demand.

The formal model utilized above can capture this insight. For example, assume that the aggregate consumption of good x in the CM is normalized such that $X^* = 1$. Nominal spending in the CM can then be written as $1/\phi_t$. Aggregate nominal spending in DM is $\alpha_t M_t$. Thus, total nominal spending is $NOM_t = 1/\phi_t + \alpha_t M_t$. Combined with the definition of nominal spending, the fact that $\phi_t M_t = z(q_t;\theta)$ implies that velocity is defined as:

$$V_t = \frac{NOM_t}{M_t} = \frac{1 + \alpha_t z(q_t;\theta)}{z(q_t;\theta)}$$

Re-arranging yields,

$$M_t \left[\frac{1 + \alpha_t z(q_t;\theta)}{z(q_t;\theta)} \right] = NOM_t \tag{10}$$

Equation (10) is the quantity theory of nominal income in which it is explicitly the case that velocity depends on the demand for real money balances in the DM, $z(q;\theta)$, and shocks to money demand, α_t. It therefore follows that fluctuations in the money supply and money demand are the primary sources of fluctuations in nominal spending/income. Given $\{M_t, \alpha_t\}$, equations (9) and (10) are a two equation system that determine $\{q_t, NOM_t\}$.[11]

[11] This two equation system is equivalent to the quantity theory of nominal income with explicit microfoundations.

3.4 Money Demand and Welfare

Based on the framework above, it is now possible to discuss the welfare implications of money demand by examining the steady-state solution to the model. Consider a steady state in which the money supply is constant. Define the socially optimal solution as that which maximizes the combined utility of buyers and sellers. As a result, the social optimization problem is to choose q to maximize $[u(q) - c(q)]$.[12] As a result, the socially optimum quantity is $q = q^*$, which satisfies:

$$\frac{u'(q)}{c'(q)} = 1$$

This condition is shown in Figure 1.

For simplicity, assume that $\theta = 1$. This assumption implies that buyers have all of the bargaining power in meetings with sellers and therefore make take it or leave it offers to the seller. These offers are just enough to compensate the seller for the cost of producing. More specifically, from equation (6), $z(q; \theta = 1) = c(q)$. With this simplifying assumption, the steady-state solution to (9) is:

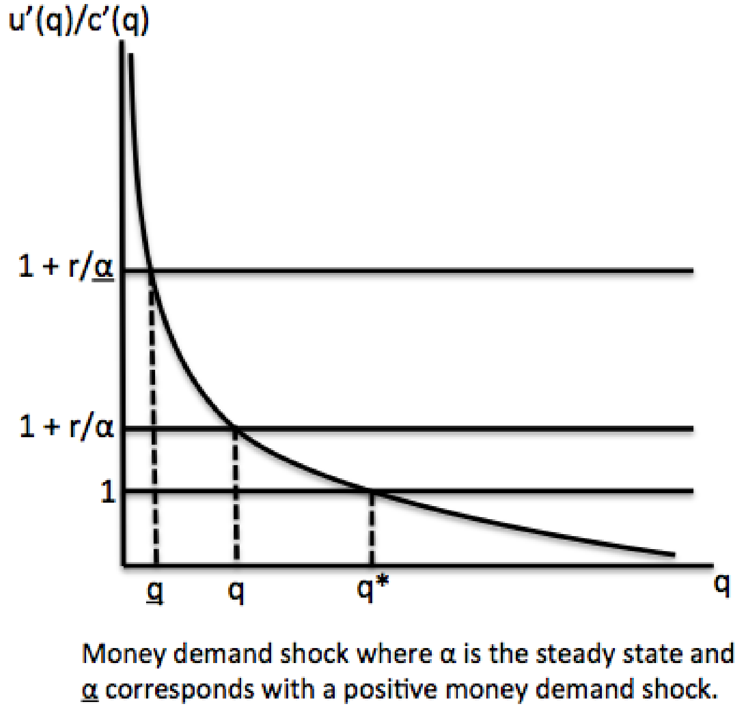
$$\frac{u'(q)}{c'(q)} = 1 + \frac{r}{\alpha}$$

where $r = \frac{1}{\beta} - 1$ is the rate of time preference. Increases in money demand make it harder for sellers to find buyers. As a result, exogenous increases in money demand result in lower values of α. As shown in Figure 1, increases in money demand reduce the amount of trade that takes place relative to the socially optimum level. As a result, this model formalizes the concept of monetary equilibrium theory in which exogenous increases in money demand are welfare-reducing in that such changes eliminate mutually beneficial trade from taking place. Note that this result has nothing to do with price stickiness. In fact, prices are assumed to be perfectly flexible.

Rather than sticky prices, the welfare-reducing effect of a money demand shock is due to the existence of decentralized markets. The sticky price assumption is only necessary if all trade is centralized in Walrasian markets. In that case, a reduction in money demand creates an excess supply of goods and services. However, the Walrasian auctioneer can instantly and costlessly adjust the price vector to restore equilibrium. In that case, there would be no need to adjust the money

[12] Since preferences are linear in x, the utility generated by the general good can be neglected from analysis.

Figure 1: Steady State Trade



supply in response to changes in money demand. However, in a decentralized market, no such coordination mechanism exists. Thus, when there is an increase in money demand, there is a decline in nominal spending and the real quantity of goods and services traded in the decentralized market.

The steady-state analysis is conducted holding the money supply is constant. Since α_t is stochastic with a constant mean and finite variance, exogenous fluctuations in money demand around the steady-state have important implications for welfare. It is possible that allowing banks to issue their own notes or having a central bank to conduct monetary policy could improve welfare. The subsequent sections, in turn, examine the implications for the money supply under a central bank and competitive note issuance.

4 The Money Supply Under Central Banking

As noted above, nominal income stabilization has often been advocated by monetary equilibrium theorists. It is therefore of interest to examine whether targeting nominal income is consistent with optimal monetary policy in the present model. The analysis below suggests that there are two broad approaches to improving welfare through monetary policy. The aggregate money demand shock reduces welfare at the extensive margin by reducing the number of trade matches. The two approaches to monetary policy work by improving trade at the intensive margin by increasing the number of quantities traded in each match. The first approach is a policy that is designed to promote the efficient steady state allocation. The second approach is to minimize deviations around the steady state.

4.1 The Friedman Rule

Typical analysis of monetary policy within the context of the search paradigm finds that the Friedman rule is optimal. For example, given the preferences outlined above, the planner's problem is to choose q to maximize:[13]

$$U = u(q) - c(q) \tag{11}$$

The solution to the planner's problem is:

$$\frac{u'(q^*)}{c'(q^*)} = 1$$

Within the context of the model above, define

$$\phi_t/\beta\phi_{t+1} = (1+r)(1+\pi_{t+1}) = 1 + i_t$$

where i is the nominal interest rate. Using this definition to solve for the steady state of equation (8) yields:[14]

$$\frac{u'(q)}{c'(q)} = 1 + \frac{i}{\sigma}$$

The optimal monetary policy is therefore to set the nominal interest rate to zero and $\mu = \beta$, where μ is the gross growth rate of the money supply.

Consider the implications of the Friedman rule in the context of monetary equilibrium. As was noted above, holding the money supply constant, an exogenous increase in aggregate money

[13]Note that since utility is linear in x_t, individuals will be indifferent across of lottery $\{x_t\}$ that delivers some given expected value.

[14]This also assumes that $\theta = 1$. If trade is conducted via Nash bargaining and $\theta < 1$, then the Friedman rule will not be optimal. There is more on this below.

demand is welfare-reducing because it reduces the number of matches and therefore the quantity of trade in the DM. In contrast, an exogenous reduction in money demand is welfare-increasing because it increases the number of matches and the quantity of trade in the DM. This latter result is due to the fact that buyers hold an inefficiently low quantity of real balances in the DM because money is costly to hold. The inefficiency that results from lower than optimal money holdings is at the intensive margin. The quantity of trade in each match is lower than the optimal quantity. Aggregate money demand shocks have implications at the extensive margin. Under the Friedman rule buyers hold a quantity of money sufficient to purchase the optimal quantity in the DM and therefore resolves the inefficiency at the intensive margin.[15]

4.2 An Alternative

Despite the optimality of the Friedman rule, there is no guarantee that the central bank will adhere to the rule. For example, Andolfatto (2010) has shown that implementing the Friedman rule through deflation is not an incentive-feasible policy when all trade is voluntary. Interest bearing money is essential. The existence of interest-bearing money, however, poses an alternative problem. In particular, the payment of interest on money balances might be prohibitively costly and therefore non-operational.[16] In addition, the Friedman rule is not optimal when there are search externalities in which the probability of finding a trading partner is dependent on the ratio of buyers and sellers in the DM (Rocheteau and Wright, 2005) or when $\theta < 1$ (Nosal and Rocheteau, 2011).[17] Regardless, the analysis proceeds under the assumption that the Friedman rule is non-operational and therefore an alternative must be chosen.

Within the present model, money demand shocks cause deviations in the quantity of the search good that is purchased relative to the steady state.[18] To the extent to which such fluctuations effect welfare, they should be minimized by the central bank. Thus, following Rotemberg and Woodford (1999), the welfare-based criteria is a second-order approximation of the utility losses experienced

[15] Aggregate money demand shocks are irrelevant. It is straightforward to show this within the context of the steady-state condition above. Intuitively, aggregate money demand shocks are irrelevant because under the Friedman rule individuals are satiated with money balances; see Niehans (1978).

[16] See White (1987).

[17] If the terms of trade are determined by Nash bargaining and the seller has some bargaining power, the terms of trade might be subject to a "hold-up" problem.

[18] This alternative policy is based on the assumption that $\mu > \beta$. Specifically, the welfare implications that follow are based on log-deviations around the zero inflation steady state.

by buyers and sellers due to deviations of in the quantity of trade relative to the steady state:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{U_t - U}{U_q q} \Big)$$

Now assume that $u(q_t) = q_t^{1-\gamma}/(1-\gamma)$, $c(q_t) = q_t$, and $\theta = 1$.[19] In the present model, taking the second-order approximation of $[u(q_t) - c(q_t)]$ and applying the expectations operator, the welfare loss can be expressed:

$$\mathbb{W} = -(\frac{\gamma\varphi}{2}) E_t \hat{q}_t^2$$

where $\varphi = q^{-\gamma}/(q^{-\gamma} - 1)$.

Then, from a log-linearization of (10), this can be re-written:

$$\mathbb{W} = -(\frac{\gamma\varphi}{2}) q^2 E_t(\hat{M}_t + \eta\hat{\alpha}_t - N\hat{O}M_t)^2 \tag{12}$$

Where $\eta = (\alpha q)/(1 + \alpha q)$.

It is now possible to evaluate whether a nominal income target is consistent with the criteria for optimal policy. Taking the nominal income target, NOM_t^*, as given the central bank can choose M_t to minimize the loss function in equation (12). The first-order condition implies:

$$M_t = NOM_t^* - v - \eta\hat{\alpha}_t \tag{13}$$

where v is steady state velocity and all variables are expressed as logs, with the exception of the money demand shock which is expressed in log-deviations from the steady state. Equation (13) implies that the central bank should set the money supply to achieve a targeted level of nominal income and that the money supply should compensate for variations in money demand. The way the policy works in the context of the model is as follows. The only shock in the model is to money demand. Increases in money demand cause a reduction in trade in the DM and a corresponding reduction in welfare at the extensive margin. The central bank offsets this welfare loss by increasing

[19] The functional form of $u(q)$ and $c(q)$ are chosen to ensure the existence of a unique steady-state; see Nosal and Rocheteau (2011). The choice of $\theta = 1$ describes a situation in which buyers make take-it-or-leave-it offers to sellers in the DM. If follows that $z(q; \theta = 1) = c(q) = q$. Lagos and Wright (2005) show that there is little reason to choose one value of θ over another because it does not alter the shape of the money demand curve (as measured by velocity) in any meaningful sense. It is shown in the Appendix that the optimality of a nominal income targeting rule is not dependent on θ.

the money supply through transfers to buyers in the DM. Since money is costly to hold, buyers who are matched with sellers spend the excess money balances thereby increasing trade in the DM. Also, it is important to note that since fluctuations in nominal income in the model are entirely determined by deviations between the quantity of money supplied and demanded, the central bank can achieve this policy even if the money demand shock is unobservable by varying the money supply such that nominal income is equal to its target.

5 The Money Supply Under Free Banking

While some have used the concept of monetary equilibrium as justification for nominal income targeting, others such as Selgin (1988, 1994) have advocated competitive note issuance on the part of private banks. This section explores the role of competitive note issuance within the context of monetary equilibrium.

So as not to alter the framework above, assume now that m denotes money in terms of bank notes.[20] These notes are issued by private banks and are redeemable in terms of base money issued by a previously existing central bank as a method of clearing payments between banks; base money does not circulate. Banks also have savings deposit liabilities. There are n banks of equal size and each bank accepts the notes of all other banks at par value. A central clearinghouse if used for the redemption of bank notes between all banks. Banks adjust their portfolio each period in which there is a total of T clearing transactions. Bank notes that come from rival banks are sent to the clearinghouse for redemption in terms of base money. While the total number of transactions is equal to T, the number of transactions that are handled by each individual bank are random.

Each bank faces a stochastic process for payments to other banks. Taking the size and composition of each bank balance sheet as given, each bank chooses the amount of reserves to minimize the cost associated with the opportunity cost of holding reserves and a penalty rate charged by the clearinghouse for lending should the bank have to borrow reserves. Formally, following Niehans

[20]For ease of analysis, in what follows in this subsection, time subscripts are neglected from analysis since the solution does involve $t + 1$ variables.

(1978) and Baltensperger (1972; 1980), the representative bank's problem is to minimize:

$$i^L \text{res} + \int_{\text{res}}^{\infty} i^p (W - \text{res}) f(W) dW$$

where W represents of withdraw of reserves due to the clearing of payments, $f(W)$ is the density function, i^L is the interest rate on loans, i^p is the interest rate paid on loans from the clearinghouse, and res denotes the reserves of the representative bank. Baltensperger (1972, 1980) shows that banks will choose to hold a positive amount of reserves if $i^L/i^p < 1/2$ and that the optimal quantity of reserves held will be proportionate to the standard deviation of W:

$$\text{res} = b\sigma$$

where b is dependent on i^L and i^p. Noting that all banks are identical this implies that the aggregate level of reserves is:

$$R = nb\sigma \tag{14}$$

Following Selgin (1994) assume that payments are determined by the following process.[21] Suppose that there is an urn that contains checks which require a transfer from one bank to another. Given each possible transaction between banks, there are n^2 checks in the urn. T_t transactions (draws from the urn with replacement) take place each period. As a result, the distribution of W is symmetrical with mean zero and standard deviation:

$$\sigma = (1/n)\sqrt{2T(n-1)}$$

Suppose that nominal income is defined as the product of the price level and real output, $NOM = PY$ and that real output is a fixed proportion of the number of transactions, $Y = \delta T$. Thus, nominal income can be expressed:

$$NOM = \delta PT \tag{15}$$

[21] This process is initially derived from Edgeworth (1888).

Assuming that the real value of each transaction is constant, the standard deviation of W can be re-written:

$$\sigma = (P/n)\sqrt{2T(n-1)} \tag{16}$$

Following Selgin (1994), equations (10), (14), (15), and (16) can be used to solve for M:

$$M = \frac{R}{b}\left[\frac{z(q;\theta)}{1+\alpha z(q;\theta)}\right]\left[\frac{\delta Y}{2(n-1)}\right]^{1/2}$$

An exogenous increase in the demand for money (a reduction in α) relative to the steady state causes a reduction in velocity and a corresponding increase in the money supply. The conclusion of the literature on free banking that the money supply should vary with money demand is therefore consistent with the present model.

6 Discussion

The assumption of price stickiness is a necessary condition for monetary disequilibrium to have significance in the context of Walrasian markets. The appeal to price stickiness often runs as follows. Since money is a medium of exchange, an excess demand for money will result in a corresponding excess supply of goods, according to Walras' Law. However, if prices are perfectly flexible, monetary disequilibrium is largely an irrelevant concept because the auctioneer can adjust prices until money demanded is equal to money supplied. As such, one way to remove this problem is to introduce firms that have enough market power to set prices, but also face costs in adjusting prices. Thus, prices will be sticky. It follows that, in the presence of excess money demand, prices will adjust only gradually such that the excess supply of goods will be persistent.

The conclusion above is dependent on the Walrasian structure of the market. In addition, as Howitt (1990: 5) notes, it "is easy to be misled by the Walrasian parable into thinking that the coordination problem for macroeconomics is equivalent to getting the prices right." Indeed, one of the features of Walrasian markets is the matching function that is also produced by the auctioneer in which economic agents are costlessly matched with one another. In addition, as noted by Clower (1975) and numerous others, the process of exchange is better considered in the context of non-centralized markets.

In the present context, it is the non-Walrasian, decentralized market that is central to the existence of money. Buyers and sellers in the model have different preferences thereby giving rise to an absence of double coincidence of wants. In addition, the trade among buyers and sellers is anonymous and therefore money is essential. As such, the model is one that dispenses with the notion of centralized markets for monetary exchange. In addition, the stochastic preference shock removes the perfect coordination of buyers and sellers. One interpretation of this shock is that buyers, in general, decide to hold a greater quantity of money balances as a store of value and therefore a smaller fraction of these buyers enter the market for trade. The smaller number of buyers entering the market, the harder it is for sellers to find buyers. This results in a corresponding reduction in trade that cannot be solved simply by adjusting prices and that is welfare-reducing. In addition, while the preference shock captured by the matching probability is stochastic in the present framework, one could easily generalize the model to include an explicit account of the decision-making process of buyers in determining whether to enter the DM. Doing so, however, would increase the mathematical complexity of the model without changing the results.[22]

Regardless of the merits of the assumption of sticky prices, the present framework demonstrates that such an assumption is unnecessary to generate the conclusions and policy implications of the concept of monetary equilibrium. The model itself, however, is agnostic about the role of sticky prices. Thus, while it remains possible to assume price stickiness in the context of monetary equilibrium analysis, it should be recognized that this is not a necessary or sufficient condition to generate the standard conclusions. To the author's knowledge, this is a novel contribution to the literature on monetary equilibrium.

7 Conclusion

The main insight of the concept of monetary equilibrium is that deviations between desired and actual money balances will, due to money's role as medium of exchange, result in changes in spending and therefore changes in nominal income. The importance of this insight is that such changes have welfare implications. As a result, monetary equilibrium theorists have often suggested

[22] This preference shock essentially amounts to a thin markets externality. Others have shown how this can arise if engaging in market activity is costly. See, for example, Howitt and McAfee (1987).

two solutions to improve welfare. The first is to target nominal income.[23] Stabilization of nominal income ensures that the money supply adjusts for changes in money demand. The second proposed solution is the idea of competitive note issuance on the part of banks as articulated by Selgin (1988). According to this view, holding the size and composition of a bank's balance sheet constant, the bank seeks to choose the level of reserves that minimizes cost. The implication is that banks will vary the amount of banknotes issued in some proportion to money demand and as a result should prevent deviations between actual and desired money balances.

This paper represents an attempt to place the concept of monetary equilibrium within the context of a formal economic model, in which money plays an important role as medium of exchange. The model extends the framework of Lagos and Wright (2005) by assuming that the matching probability in the decentralized market can be interpreted as a stochastic money demand shock. Intuitively, this shock can be thought of as a money demand shock because increases in money demand (reductions in the matching probability) mean that it is harder for sellers to find buyers and there is a reduction in spending; and correspondingly for reductions in money demand.

This framework is subsequently used to evaluate the behavior of banks under competitive note issuance and to examine the role for monetary policy. Regarding the former, the model incorporates the behavior of banks used in Selgin (1994) to demonstrate that under competitive note issuance, banks would vary the notes in circulation with money demand. With regards to the latter, the paper develops a welfare criteria based on Rotemberg and Woodford (1999). Specifically, welfare losses are defined as the expected loss in utility caused by deviations from the steady-state. Taking a second-order approximation of the sum of the utility of buyers and sellers. This implies that welfare losses are represented by squared deviations of the quantity of the search good traded relative to the steady state. It is then shown that the central bank can choose the money supply to minimize the loss function and that a nominal income target is consistent with optimal policy. These results suggest that the conclusions of monetary equilibrium theory are both sound and independent of assumptions about price stickiness.

[23]This conclusion explicitly follows from Friedman's (1974) quantity theory of nominal income in which fluctuations in the supply and demand for money are the primary source of changes in nominal income.

Appendix

A.1 Money Demand

The model presented in this paper interprets the stochastic matching parameter, α, as a money demand shock. It will now be shown that α is isomorphic to a money demand shock in a model with money in the utility function.

Suppose that utility for an infinitely lived, representative consumer is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{{c_t}^{1-\gamma}}{1-\gamma} + \varepsilon_t \frac{{m_t}^{1-\Psi}}{1-\Psi} \tag{17}$$

where c is consumption, m is real money balances, and ε is a stochastic preference parameter attached to real balances. The representative agent enters period t with money balances, M_t, bond balances, B_t, and receives a transfer from the central bank, T_t. During the period, the representative agent earns income, Y_t, and uses wealth and income to finance consumption, c_t, and allocate the remainder to bond and money holdings. As a result, the representative agent's budget constraint is:

$$m_{t+1} + b_{t+1} + y_t = m_t + (1 + i_t)b_t + c_t + T_t$$

where i_t is the nominal interest rate and real values are expressed as lower case letters.

Maximizing (17) subject to the budget constraint yields the following first-order conditions:

$$c_t^{-\gamma} = \lambda_t$$

$$\lambda_t = (1 + i_t)\beta\lambda_{t+1}$$

$$\varepsilon_t m_t^{-\Psi} = \lambda_t - \beta\lambda_{t+1}$$

Combining the three first-order conditions yields:

$$\varepsilon_t m_t^{-\Psi} = c_t^{-\gamma} i_t \tag{18}$$

In equilibrium, $M_t + T_t = M_{t+1}$ and $B_t = B_{t+1} = 0$. Thus, from the budget constraint, $y_t = c_t$.

Finally, assume that money demand is unit elastic with respect to income ($\Psi = \gamma$). Then, one can re-write (18) as:

$$V_t = f(i_t, \varepsilon_t) = \left(\frac{i_t}{\varepsilon_t}\right)^{\frac{1}{\Psi}}$$

From the search model, velocity is:

$$V_t = f(i_t, \alpha_t) = \frac{1 + \alpha_t z(q_t; \theta)}{z(q_t; \theta)}$$

where velocity is a function of i through $z(q;\theta)$. Define $\alpha_t \varepsilon_t = e$, where e is some arbitrary constant. It follows that velocity in both the MIU and search framework is an increasing function of i and α.

A.2 Nominal Income Targeting Under Alternative Bargaining Conditions

In the discussion of nominal income targeting, it was assumed that $\theta = 1$ (i.e. buyers determine prices in the decentralized market). Here it is demonstrated that the assumption that $\theta = 1$ substantially simplifies the mathematics, but not alter the main results. To do so, three alternative bargaining conditions are examined. The first example considers policy under Nash bargaining. The second example considers policy in which sellers determine prices (i.e. $\theta = 0$).[24] In the final example, both the buyer and the seller receive a fixed proportion of the surplus. The examples illustrate that the terms of trade in the decentralized market are of little consequence when the objective is to minimize fluctuations of trade in the DM around the steady state.

A.2.1 Policy Under Nash Bargaining

Consider equation (10), where $z(q;\theta)$ represents the demand for money when the terms of trade in the decentralized market are determined through Nash bargaining between buyers and sellers. In the text, it was assumed that buyers make take-it-or-leave-it offers to sellers (i.e. $\theta = 1$). Given the functional forms in the model, this implied that $z(q;\theta) = c(q) = q$. Here, we dispense with this assumption to demonstrate that while the assumption of $\theta = 1$ greatly simplifies the mathematics, it does not alter the conclusion.

[24] It might be more intuitive to consider this case as referring to the limit as θ goes to zero, since buyers might choose not to trade if they don't receive any of the surplus.

Log-linearizing (10), yields:

$$\xi \hat{q}_t + \eta \hat{\alpha}_t = N\hat{O}M_t - \hat{M}_t$$

where

$$\xi = \frac{-\gamma\theta\left[\theta + \frac{1-\theta}{1-\gamma}\right]q^{1-2\gamma} - [\theta q^{-\gamma} + 1 - \theta][(1-\gamma)\theta + (1-\theta)]q^{1-\gamma}}{\left\{\left[\theta + \frac{1-\theta}{1-\gamma}\right]q^{1-\gamma}\right\}^2}$$

and

$$\eta = \frac{\alpha}{\frac{\theta q^{-\gamma}+1-\theta}{[\theta+\frac{1-\theta}{1-\gamma}]q^{1-\gamma}} + \alpha}$$

Plugging this log-linearized equation into the loss function yields:

$$\mathbb{W} = -\big(\frac{\gamma\varphi}{2}\big)(1/\xi)^2 E_t(\hat{M}_t + \eta\hat{\alpha}_t - N\hat{O}M_t)^2$$

It is straightforward to see that optimal policy is consistent with the rule in equation (13), albeit with a different definition for η.

A.2.2 Policy When Sellers Determine the Terms of Trade

Now consider the case in which $\theta = 0$. In this case, from (6), it is true that:

$$z(q;\theta = 0) = u(q)c'(q)$$

Given the functional forms in the text, this implies that

$$z(q;\theta = 0) = u(q) = \frac{q^{1-\gamma}}{1-\gamma}$$

Log-linearizing (10) using this definition for money demand yields,

$$\upsilon \hat{q}_t + \eta \hat{\alpha}_t = N\hat{O}M_t - \hat{M}_t$$

where

$$\upsilon = \frac{(\gamma-1)(1-\gamma)q^{\gamma-1}}{(1-\gamma)q^{\gamma-1} + \alpha}$$

and

$$\eta = \frac{\alpha}{(1-\gamma)q^{\gamma-1} + \alpha}$$

Plugging this in to the loss function yields

$$\mathbb{W} = -\left(\frac{\gamma\varphi}{2}\right)(1/\upsilon)^2 E_t(\hat{M}_t + \eta\hat{\alpha}_t - N\hat{O}M_t)^2$$

Again, it is straightforward to see that optimal policy is consistent with the rule in equation (13), albeit again with a different definition for η.

A.2.3 Policy Under a Proportional Solution

Suppose that buyers and sellers each receive a fixed proportion of the surplus. Denote the fraction of the surplus that goes to buyers as θ and the fraction that goes to sellers as $1-\theta$. In terms of the model, this implies that

$$u(q) - \phi m = \theta[u(q) - c(q)]$$

and

$$-c(q) + \phi m = (1-\theta)[u(q) - c(q)]$$

Combining this conditions and solving for ϕm yields a money demand equation:

$$\phi m = (1-\theta)u(q) + \theta c(q) \equiv z(q)$$

Plugging this expression for z into (10) and log-linearizing yields:

$$\nu\hat{q}_t + \eta\hat{\alpha}_t = N\hat{O}M_t - \hat{M}_t$$

where

$$\nu = \frac{-\theta q + (1-\theta)q^{1-\gamma}}{\frac{1}{\theta q + \frac{1-\theta}{1-\gamma}q^{1-\gamma}} + \alpha}$$

and

$$\eta = \frac{\alpha}{\frac{1}{\theta q + \frac{1-\theta}{1-\gamma}q^{1-\gamma}} + \alpha}$$

Plugging this in to the loss function yields

$$\mathbb{W} = -\big(\frac{\gamma\varphi}{2}\big)(1/\nu)^2 E_t(\hat{M}_t + \eta\hat{\alpha}_t - N\hat{O}M_t)^2$$

Again, it is straightforward to see that optimal monetary policy is consistent with the rule in equation (13), albeit with a different definition of η.

A.2.4 A Note On Optimal Policy Under Alternative Terms of Trade

While it is clear that the above bargaining solutions are consistent with the monetary policy rule in (13), a necessary condition for these rules to be consistent with monetary equilibrium is that $\eta > 0$. When the terms of trade are determined in the three ways described above, the functional form of the buyers utility function is of some consequence. Nonetheless, given the general functional forms used in the main text, a sufficient condition for $\eta > 0$ is that $\gamma < 1$. Whether this condition holds is therefore an empirical question. Lagos and Wright (2005) use the same functional forms as the present paper and choose the parameters of the model to fit a money demand function. They find that the best fit of the model is consistent with this condition. In fact, the highest value they obtain to fit the model is $\gamma = 0.266$ and this is not dependent on θ.

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